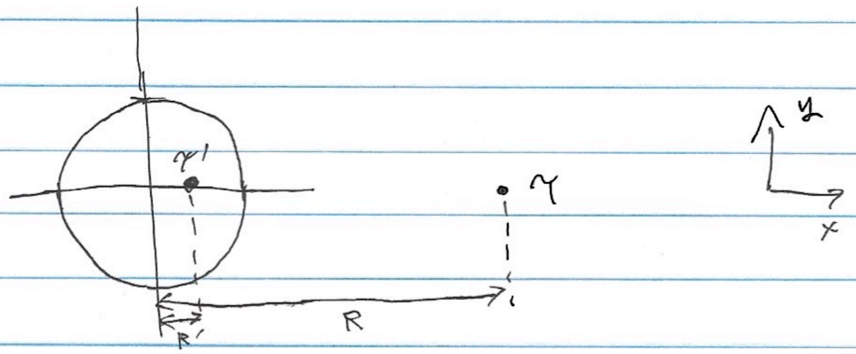
Jackson 2.11

(a)



First use image line to ground the cylinder.

$$\Phi(r,\phi) = k\left[\frac{\tau'}{\sqrt{R'^2+r^2-2R'r\cos\phi}} + \frac{\tau}{\sqrt{R^2+r^2-2Rr\cos\phi}}\right]$$

Demanding $\Phi\big|_{r=b} = 0$ gives

$$\frac{\tau'}{\sqrt{R'^2+b^2-2R'b\cos\phi}} + \frac{\tau}{\sqrt{R^2+b^2-2Rb\cos\phi}} = 0.$$

At $\phi=0$, $\phi=\pi$ gives two separate equations

$$\begin{cases} \dfrac{\tau'}{\pm(R'-b)} + \dfrac{\tau}{\pm(R-b)} = 0 \\[2ex] \dfrac{\tau'}{R'+b} + \dfrac{\tau}{R+b} = 0 \end{cases}$$

The system of equations for opposite signs gives nontrivial soln

$$\boxed{R' = \frac{b^2}{R}, \quad \tau' = -\tau\frac{b}{R}}$$

It remains to put cylinder at potential V, use line of length $2L$, charge density λ, $L >> b$.

$$\Phi(r=b) = k \int_{-L}^{L} \frac{\lambda}{\sqrt{b^2+x^2}}\, dx$$

$$= k\lambda \left[\ln (x + \sqrt{b^2+x^2}) \right]_{-L}^{L}$$

$$= k \lambda \ln \left[\frac{\sqrt{b^2+L^2} + L}{\sqrt{b^2+L^2} - L} \right].$$

$$\Phi(r=b) = V \implies \lambda = \frac{V}{k \ln\left[\frac{\sqrt{b^2+L^2}+L}{\sqrt{b^2+L^2}-L} \right]}$$

expanding $\sqrt{b^2+L^2}$: $L\sqrt{1+\left(\frac{b}{L}\right)^2}$

$$\simeq L\left[1 + \frac{1}{2}\left(\frac{b}{L}\right)^2 + \cdots \right]$$

$$\simeq L + \frac{b^2}{2L} + \cdots$$

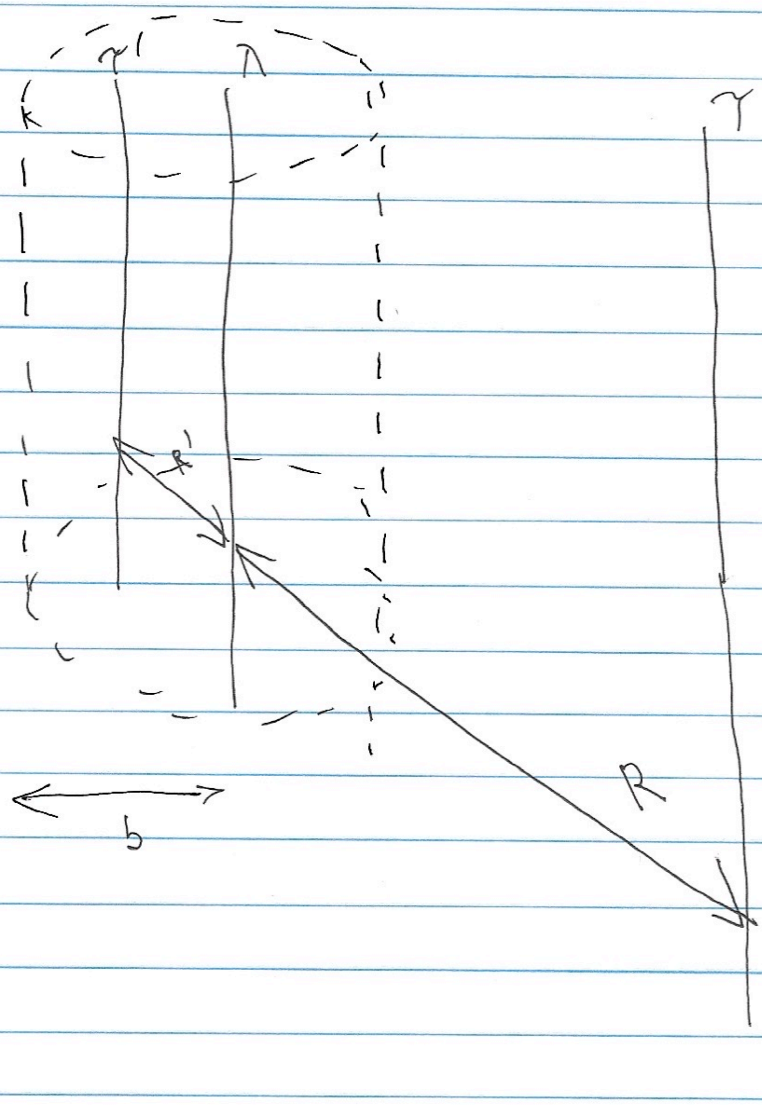
$$\implies \frac{\sqrt{b^2+L^2}+L}{\sqrt{b^2+L^2}-L} \simeq \frac{2L + \frac{b^2}{2L}}{\frac{b^2}{2L}} = 1 + \frac{2L\, 2L}{b^2}$$

$$= 1 + \left(\frac{2L}{b}\right)^2$$

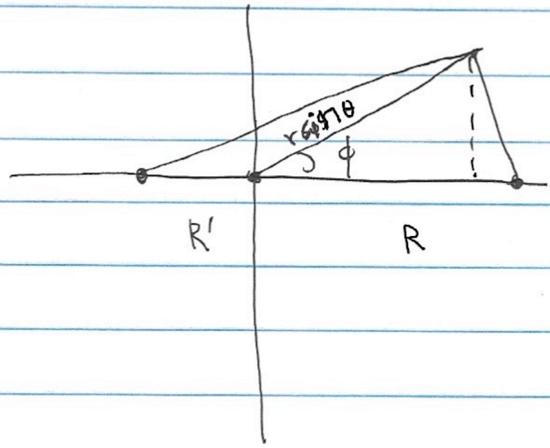
$$\implies \ln\left[\frac{\sqrt{b^2+L^2}+L}{\sqrt{b^2+L^2}-L} \right] \simeq \ln\left[1 + \left(\frac{2L}{b}\right)^2 \right].$$

$$\Rightarrow \Phi(r=b) \approx k\lambda \ln\left[1+\left(\frac{2L}{b}\right)^2\right]$$

and this gives for $\Phi(r=b) = V$, $\lambda \approx \dfrac{V}{k\ln\left[1+\left(\frac{2L}{b}\right)^2\right]}$



(b) Only the separation between the point of interest P and the line charges in the x-y plane is relevant.



$$\Phi = k\lambda \ln\left[1 + \frac{4L^2}{r^2 \sin^2\theta}\right] + k\lambda' \ln\left[1 + \frac{4L^2}{R'^2 + r^2\sin^2\theta + 2R' r \sin\theta\cos\phi}\right]$$

$$+ k\lambda \ln\left[1 + \frac{4L^2}{R^2 + r^2\sin^2\theta - 2R r \sin\theta\cos\phi}\right]$$

with λ', R' given in (a).

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